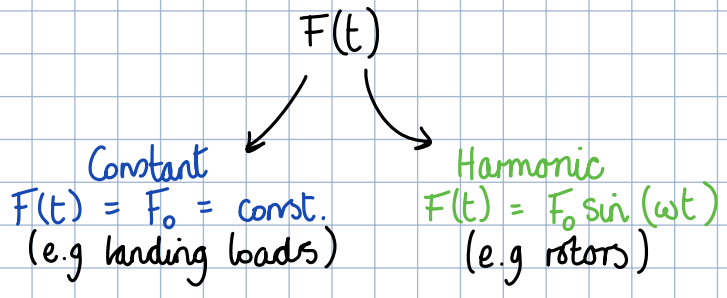
Forced response of 1 DOF system:

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

→ must solve a non-homogenous ODE:

→ solve homogenous ODE: component due to free response
→ solve rhs: component due to an applied force F(t)

Total solution = Complimentary solution + Particular solution



$$\text{Total solution} = x_H + x_p = e^{-\zeta\omega_0 t}\left(X_1 \sin(\omega_0 t) + X_2 \cos(\omega_0 t)\right) + x_{forced}$$

↳ solutions can be superimposed as it's a linear system

Particular Solution for Constant Force: $F(t) = F_0$

- Guess trial solution as a constant: $x_p = C$

→ substitute trial solution into EOM:

$$m(0) + c(0) + kC = F_0$$

$$\therefore \quad C = \boxed{x_p = \frac{F_0}{k}}$$

Therefore total solution is: $x = e^{-\zeta\omega_0 t}\left(X_1 \sin(\omega_0 t) + X_2 \cos(\omega_0 t)\right) + \frac{F_0}{k}$

Complementary solution from initial conditions

static deformation due to constant force F_0

Constant force produces steady-state static deflection (new equalibruim position).